Chapter 4

Problem-Solving Classes

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INTRODUCTION

In many important respects, tutorials in the sciences share similar characteristics to their counterparts in the humanities and social sciences. Across this spectrum of disciplines for example, tutorials provide an invaluable link between the concentrated delivery of material in lectures and students' private study, while also giving students opportunities to work interactively with their peers and with a tutor. Equally importantly, tutorials in all of these subject areas enable tutors to contribute their insights and understandings without denying students the chance to take responsibility for their own learning.

Yet there are also differences which sharply distinguish tutorial practices across the arts–science continuum. Perhaps the most fundamental of these differences stems from the nature of the course material. In mathematics, the natural sciences and engineering, the subject matter is characterised by strong logical threads, which in turn leads to a fairly strict sequential presentation. Students are exposed to a hierarchical structure or a series of stepping stones, where failure to understand one topic can prejudice their chances of coping adequately with later topics which build on it. Hence, an individual with a 'mental block' on a topic, especially one near the start of the course, can quickly land in a perilous state.

The present chapter focusses on tutorials of this kind, dealing particularly with *problem-solving* tutorials. The examples presented are drawn from mathematics, where, it can be argued, the purest form of problem-solving is to be found. As defined here, problem-solving is taken to mean any exercises set for students to solve. In mathematics these exercises are usually *objective* in nature. The problems are tackled by a *solution* method which is often either correct or incorrect and, if there is an *answer*, it is often either right or wrong. This distinction is a crucial one: some problems, such as 'what is 2+2?', have an *answer*, while others, such as 'show that 2+2=4' have a *solution*. Furthermore, the working which leads to an answer is also treated as an integral part of the solution. In other forms of problem-solving, this is not necessarily the case. In data analysis or experimental design, for example, there may be no single correct solution but rather a spectrum of possible answers with some more valid than others. However, in both cases the objective nature of the subject matter strongly determines the content of problem-solving classes and the teaching approach followed.

The material in this chapter has been structured to guide new tutors as they pass from appointment, through training and preparation, to conducting regular problem-solving sessions with their groups. Tutors are invited:

- to review what is meant by problem-solving in the light of recent research;
- to reflect on their own abilities and assumptions about problem-solving;
- to acquaint themselves with the purposes tutorial sessions serve in their discipline;
- to consider in detail the way first and later sessions are prepared for and managed;
- to establish good practice in the marking of submitted coursework.

Throughout, an assumption is made that the tutor will be working in the context of a small group. Some courses may hold problem-solving classes with a large group of students in one room, with several tutors in attendance; much of what follows still applies, suitably scaled-up, but there is clearly little opportunity for group discussion where classes are very large.

PROBLEM-SOLVING

A great deal of research, principally by the artificial intelligence community, has been devoted to discovering how people solve problems and, as a corollary, to find methods to improve problem-solving skills. Much of this has involved mathematical problems, which avoid extraneous distractions, although some of the ideas which have evolved have been applied to other science disciplines. Indeed, certain of the phenomena have been detected in areas outside academia, such as the business environment. Some of the salient points are presented below, as a complement to chapter 8, *Student Learning* .

Prior to this work, the key categories in problem-solving were thought to be factual knowledge (resources) and methods for applying that knowledge (heuristics). The most influential contribution, by Schoenfeld[1], introduced two further categories, the strategy which guides use of resources and heuristics (control) and the solver's attitudes (belief). These four categories are expanded and illustrated in *figure 1*, and some further comments are made below.

Resources

The delivery of these is primarily the job of the course team, but the securing of them, and the eradication of 'bugs', i.e. systematic flaws in their use, are important responsibilities for tutors.

The course team will undoubtedly want students to develop a sound technique in standard problems, and will see private study, backed up by tutorial provision, as the best way to ensure it. There is evidence to support such an aim[2]. It appears that we have the ability to process information 'automatically'. Since students are often overwhelmed by the number of processes they need to control when tackling a novel problem, it helps when they can switch some processes to 'automatic'. A useful analogy is riding a bicycle. Learning to do this is a daunting task because of the number of different factors which must be controlled. An experienced rider does not have to think about balance or pedalling and can concentrate on steering, signalling, pacing, etc.

Although resources are clearly of critical importance, investigators have found students unable to solve problems for which they already possess the necessary resources and more! Scanlon[3] reports physics students who understand concepts but cannot solve problems involving them.

PROBLEM-SOLVING CATEGORIES

Resources

- knowledge about the problem domain
- facts, data and definitions
- ability to execute algorithmic procedures
- ability to carry out routine procedures
- knowledge of rules of discourse

Heuristics

- use an analogy
- draw a figure or graph
- try some special cases
- identify sub-goals
- try working backwards
- derive something from the data
- identify a related problem
- make use of symmetry

Control

- identify the key features of the problem
- consider all relevant methods before starting detailed work
- monitor solution process
- assess validity of intermediate solutions
- be prepared to switch method
- do not commence complicated analysis unless it is clearly unavoidable

Belief

- unquestioned intuitive knowledge, possibly incorrect
- use of a particular heuristic, such as a diagram, for every problem
- belief that solution is correct, even when palpably wrong
- decision that something is ‘obvious’ when, in fact, it is false
- ‘I can never solve such problems’
- ‘That sort of problem is always solved using this method’
- ‘All these problems can be solved by a slick method’
- ‘These problems are always harder than they appear’
- ‘This material is of no practical use’

Figure 1

Heuristics

Following the publication of Polya's classic book, *How To Solve It*[4], it was thought that resources merely needed to be complemented by a full list of heuristics. The programme to identify and use these has proved disappointing because of the sheer number, especially when some have to be subcategorised; Schoenfeld[1] lists five problems, each of which needs a quite different application of the heuristic 'Try some special cases'.

By their nature, heuristics tend to apply across a range of individual courses and so are rarely taught formally. This places responsibility on tutors to help students construct and develop an effective armoury. Although they are undoubtedly very important, they cannot replace inadequate resources.

Control

The large number of useful heuristics highlights the problem of choosing the best strategy for a particular problem. Knowing a heuristic is not enough; the student must also know when to use it.

Even when students are presented with a set of well-defined methods, examinations often uncover uncertainty when the problems are mixed, even when they appeared to have no such difficulty during term-time, when the choice of method could be deduced from the context. Students need to think carefully about the best approach, not only to choose a feasible one, but to choose an efficient one. Having chosen a strategy, they must be prepared to switch to an alternative as soon as further progress seems impossible.

These so-called 'higher-order skills' – circumspection, monitoring and flexibility – are key features of successful problem-solving. Researchers have reported phenomena such as students considering candidate methods in the order in which they were taught[1] and apparently choosing a method on the grounds that it was the only one not yet used in that set of exercises[3]. Much research has been done by comparing experts and novices to see if the former can be role models for the latter. Silver[2] reports that experts tend to think qualitatively about problems before engaging in the detailed solution, while novices tend to start quantitative work immediately. A study in physics, where solvers were asked to identify key features of problems, showed that novices would give 'surface' answers, such as pulley, inclined plane, spring, while experts gave 'deep' answers, such as Newton's third law, conservation of energy. (The use of the terms 'deep' and 'surface' recalls work reported in chapter 8, and tutors will find several reflections of that dichotomy in the analysis reported here.)

Tutors must try to help students in this. Silver[2] remarks that textbooks, although excellent for presenting resources, are not good media for inculcating control.

> The demonstration and modeling needs to focus not only on *what* is being done but also on *why* the choice was made.

In tutorial work students should certainly be warned off an infeasible method, to save their time, but tutors ought to avoid telling a student directly the optimum method. Thinking about strategy may be part of the exercise, and hints, by means of a few carefully phrased questions, are more likely to preserve that. Entwistle[5] found students critical of very large problem-solving classes where they were unable to seek individual help. In these, students were shown the 'correct' solution, encouraging surface learning.

Belief

In some ways an unhelpful ingrained belief system is the most pernicious obstacle facing the novice problem-solver. Tutors must ensure they do not, by careless comments, further reduce the confidence of students who believe they are incapable of solving problems of the sort covered in the course. Also, they should bear in mind that, in spite of the initial aim of much of the research in this area, there is evidence[3] that experts may not be good role models, their performance being too 'opportunistic'. On a more positive note, they should encourage students to shed misconceptions about how problems are solved, about which method is 'always' best for a type of problem and about the usefulness of the material. It may be necessary to explain the need to ignore a 'world-view' when the material is counter-intuitive, as in some formal physics[1].

Thus, although intuition can be a powerful tool in determining a viable approach to a problem, it must not be applied unquestioningly. Encouragement to adopt the deeper approach discussed under 'Control', by stepping back from the problem and keeping an open mind about both the problem and the best strategy, ought to help develop students' skills.

Does Schoenfeld's analysis correspond to problem-solving as you see it in your discipline?

What mental processes do you use to solve problems?

Are there beliefs that you hold which may constrain your own problem-solving abilities?

Implications for Learning

The special nature of problem-solving leads to differences in the way tutorials are run, compared with those described in chapter 3. The principal ones may be summarised as follows:

- there is likely to be more individual or subgroup work and discussion, and less group discussion;
- there may be more emphasis on consolidation of the material than on extension, although this varies from course to course - this is especially likely in so-called 'service-teaching', where the course is a compulsory, but not mainstream, part of the students' curriculum;
- there will be more emphasis on logical and analytical thinking than on critical thinking;
- expression and argument are more likely to be expressed in writing;
- tutors may have less freedom to decide the content of classes – other staff may wish to tune the activity to the hierarchical structure they are building in their courses;
- there may be a wider range in students' ability to contribute to and gain from the tutorial session, although this may depend on the way students have been allocated to the different groups.

What then are the practicalities to attend to? What is it important to know about the day-to-day work of a tutor preparing to take problem-solving classes?

GROUNDWORK

It is often the case that tutorials form part of a teaching package. The basic course material will be designed and presented in lectures by a course team, headed by a course leader. Part of the responsibility of the latter is to ensure that tutorial staff are well-briefed about the course objectives and their part in achieving them. Tutors' contracts should contain the name of the course leader for each relevant course, and tutors should feel prepared to approach that person if they require help or support.

Tutors who deal with problem-solving classes often have their duties prescribed in a fair amount of detail. There are three common ways in which they receive guidance about this.

Student Information

Tutors should be provided with the same information booklets or sheets as the students in the course. These should be studied in detail as they are likely to set out course aims and objectives, recommended textbooks, assessment information and other helpful organisational details. Indeed, it is not uncommon for tutors to be asked questions by their tutees which can be answered by reference to this! Care should be taken, however, not to stray beyond this, for example in responding to questions such as "Will I get an exemption from the examination?" which are a matter for the course leader.

Initial Briefing

It is usual for tutors to receive briefing before the course commences, concerning items such as the backgrounds of the students in the various groups, the method of marking coursework and perhaps even a preferred plan for tutorial sessions. This is best provided by means of a meeting of the course team and tutors, backed up by written instructions. If, through pressure of time, a meeting of all tutors proves impossible, course leaders will try to meet tutors individually, especially those new to the course. At this stage, tutors should ensure instructions cover foreseeable questions. This includes the problems which can arise in the delicate area of assessment of coursework where it is essential that all tutors adopt the same rules.

Regular Briefing

Tutors should receive instructions and suggestions for the material to be covered in each session. The more the course team desires to prescribe the work of the session, the more detailed this is likely to be. A specimen instruction sheet, for a mathematics course, is set out in *figure 2*. Such information should be helpful in showing what material is to be mastered, although probably less on how.

Note the portion at the foot of this sheet, which invites tutors to feed back information about the

SAMPLE TUTORS' ADVICE SHEET

Mathematics 2Eh

Tutorial Date: 22nd November

Material Covered in Lectures:

The use of differentials to deal with implicit functions, particularly 'equations of state', such as those which arise in thermodynamics. (Note that this is less important for the groups which meet at 10am.) Determination of stationary points, mostly for functions of two variables, and their nature. Constrained problems, solved using Lagrange multipliers. The solution of the $\frac{\partial L}{\partial x_i} = 0$ equations for Lagrange multiplier problems is less important than learning how to define L and how to interpret the λ_i values.

Hand-in Questions:

Worksheet 6; Exercises 12.4 and 13.2

Suggested Tutorial Work:

The stationary point material is the most important. The theory is fairly straightforward, and students usually find more difficulty in manipulating the resulting equations. 13.1 and 13.5 are both standard problems and good for practice in solving the equations. 14.2 is a good constrained problem, since it is a compromise between being sufficiently general and being too complicated.

Solutions to be given to the group: 12.1–14.3

Next Week's Hand-in: Worksheet 7; Exercises 15.2 and 16.1

Tutor's Report Section

Students giving rise for concern (with reasons):

Smith has now been missing for two weeks. Brown handed in very poor work, and has told me he is struggling in this topic. I gave him some pointers to help him back on the rails, but I fear he may need more direct help.

Students previously reported, now improving:

Jones, who missed the previous two sessions, has now reappeared. He has been ill, and I gather his Director of Studies will contact the Department.

Other comments for Lecturing Staff:

Most students found the solution of the Lagrange equations difficult, and would like some guidance on how important this is for examination purposes.

Tutor signature: E Bloggs

Figure 2

students in the group and the course material. Tutors not invited to feed back information in this way should not, however, feel inhibited about conveying comments to their course leader.

Coursework – Early Thoughts

Throughout this chapter it is assumed that students will have submitted material for marking prior to the meeting of the group, and that it will be the duty of the tutor to return the material, and perhaps also to mark it. (If that is not the case, the reader may safely skip the parts below which cover coursework.) Although guidance on marking will be given at the end of the chapter, it is useful to address here the various reasons why course teams require coursework to be submitted, since it may affect the way tutorial sessions are run. There are several reasons, some or all of which may be relevant for a particular course:

- it encourages students to keep pace with material – if the material is structured hierarchically, it is particularly difficult to catch up and grasp new material at the same time;
- it can be diagnostic, helping students to assess their progress, tutors to assess where help is particularly needed and the course team to assess the success of the teaching;
- it may be instructional, for example using a problem the solution to which enhances the students' grasp of the subject;
- it often provides a contact point between the tutor and the group or even individual students - it can suggest where tutorial discussion should commence;
- it may contribute to the overall assessment.

These reasons indicate some very beneficial features of coursework as far as the tutor is concerned. Care must be taken, however, if there is an assessment role. The tutor should try to emphasise the positive aspects, and to relegate his or her role of allocating marks as merely acting as an agent of the course team, under its instructions. Fairness in marking and openness when questioned about decisions will help in this. Tutors who feel apprehensive about this are urged to seek early guidance and advice from experienced staff.

Looking ahead to your tutorial classes, can you identify any issues for which you would welcome clarification?

TAKING TUTORIAL CLASSES

The primary aim of the tutorial sessions is to encourage the development of students' capacities to apply material more formally delivered in other components of the course, and to extend their studies to fresh material, albeit based on what has already been presented. This is implemented by giving each member of the class access to a person of appropriate academic standing. This is in marked contrast to lectures, where the objective is essentially to deliver fresh material to the students, and the amount of contact between lecturer and individual class members is minimal. Because the tutorial is a timetabled point of contact between staff and students, great importance is attached to its success.

The content of this chapter is based on the premise that the tutor's role is not only to be a source of academic expertise but also to encourage members of the group to seek and draw on that expertise. In order to give encouragement, however, the tutor must attempt to develop a suitable atmosphere in the tutorial. The tutor should aim to be perceived by the group as a 'friendly guide', who is interested in the welfare of his or her tutees (primarily their academic welfare, but also more than this if it helps the working relationship without unduly burdening the tutor); who is keen for members of the group to be successful academically and who is seen to be keen in this way; who will go to considerable lengths to sort out the difficulties presented to him or her by the tutees, and will also attempt to communicate his or her own enthusiasm for the subject to the tutorial group. Nonetheless, it is advisable to avoid undue camaraderie, lest the tutor becomes unable to maintain 'ground rules' (see later) such as prompt arrival and timely submission of coursework.

In trying to achieve such an atmosphere, there are a number of practical steps which the tutor can take (detailed in later sections); equally, in dealing with the minute-by-minute cut and thrust of a session, the tutor has to rely on a combination of experience and intuition for which it is difficult to formulate rules other than to try to adhere to the general principles presented here.

As explained above, there is likely to be more emphasis on individual than on group teaching. The group size is likely to represent a compromise between the desire to make it as small as possible, thereby maximising contact between tutor and individual students, and the resources available. The conduct of a 'small' group, say with 12 or less

students, will depend on the attitude of each participating person, the students no less than the tutor.

We make an implicit assumption that the tutor will stay with a group for a significant period. Some courses have a more fragmented structure; also, on occasion a tutor may have to act as a 'stand-in'. In such instances course leaders have to develop long-term plans and tutors can help them by briefing those who will take over when they have completed their own stint.

Content of Session

Course Objectives. Some courses may have very concrete objectives, such as achieving the ability to solve specific problems. The problems set are likely to be those with a specific answer. Students will not necessarily judge that the style and presentation of the solution are as important as the correctness of the answer! Tutorial help is then likely to revolve round the abilities of individuals to achieve the objectives. This situation is most common in 'service' courses, such as mathematics courses for engineers and scientists.

On the other hand, some courses may have more open-ended objectives, such as achieving an understanding of some body of knowledge. The problems set may require the student to demonstrate the truth of a result, with the style of argument and its lay-out being of great importance. Here there is more scope for discussion, with the tutor helping elucidate key features and possibly links with other material. This situation tends to occur in courses where students intend to proceed to Honours in the discipline, particularly in the later years of the course when they have gained in confidence.

Group Profile. Students may be allocated to groups according to one or more of the following factors:

- home department (for 'service' courses);
- background (H-grade, CSYS, A-level);
- demonstrated ability (grade in school examination, performance in previous university course);
- different learning types - see chapter 8, *Student Learning* – such a classification is currently rare, largely because of the difficulty in implementing it.

Whether or not such allocations are good practice, they may be impossible to achieve for various reasons; tutors must then be prepared for an inhomogeneous group, which may present delicate problems in balancing time. It is recommended that tutors ask the course leader about the group background if such information has not been divulged.

Coursework. As explained in the next section, most courses set work to be done privately and handed in for comment and possibly assessment. Where marking is undertaken by the tutor, the work is usually returned in the tutorial session, which clearly affects the running of the session. There is a delicate issue here; coursework is an invaluable diagnostic instrument for both student and tutor, yet there is an obvious danger that the tutor's role may be undermined if he or she is regarded primarily as an 'assessor'. If any grades allocated are to count towards the overall course assessment, tutors should strive to be tactful in any comments made, whether spoken or in writing. (Some advice on written comments is given towards the end of the chapter.)

POSSIBLE ACTIVITIES

- review of coursework
- tutor-led discussion
- student-led discussion
- worked example (by tutor)
- worked example (by student)
- group work
- individual work (solo)
- individual work (in group)

Figure 3

We have already mentioned that there are few general rules for what must be contained in a specific session. Some possible activities for the session are set out in *figure 3*. These will be discussed in more detail later. The decision as to which of these to employ at any time may be determined by numerous factors, such as prescription by the course team, student choice, tutor suggestion and the course of events within the session.

Preparing Students

It is important to stress the need for students to prepare for each session. Tutors should set an example by being well-prepared and aware of the context of the tutorial and the learning aims associated with it.

Students ought to have been told that the success of tutorials depends on their willingness to contribute to the proceedings, and that success in this is largely achieved through preparation before the tutorial, which will inevitably identify difficulties, and thus generate points of query for the student to bring up at the tutorial.

In practice, there is usually variation in the willingness of students to do adequate preparation or to contribute to the tutorial. It is a further aim, for which the tutor is largely responsible, to encourage preparation by showing how it improves the success of the session.

To this end, tutors could consider suggesting problems to be tackled before the session, unless that is already done by members of the course team. Such problems should be fairly straightforward to ensure students are not discouraged; after all, they will be tackling them without tutor help.

It is sensible to make it clear to the group that it can only function as a coherent unit if everyone does the same preparation, which is another reason for setting tractable problems. If some special activity is planned, such as a student presentation, then the entire group should be briefed; a suitable hand-out could be prepared in advance.

Recall that the session interfaces both with the lectures and the students' private study, which indicates two main aims – providing back-up for material formally delivered to the class as a whole and helping students' individual attempts to come to terms with that material. Their preparation should enable them to benefit from help in both directions.

Tutorial sessions are also effective in engendering a spirit of group-work, even if that is not an explicit course objective. If students can achieve fruitful collaborative work in a problem-solving class, there is a good chance that they will continue to work in that way for a proportion of their private study time. Provided there is true collaboration, this is an effective and efficient method of developing problem-solving skills.

Detailed aims for tutorial sessions are strongly influenced by the discipline, both in what knowledge and abilities students are expected to gain, and in the way they are to work towards gaining them. A similar statement can be made about the sort of preparation which students should carry out.

Given the particular tutorial circumstances you expect, what strategies do you think will prove most effective in encouraging students to undertake preparation?

THE FIRST SESSION

It is now time to consider activities in the actual sessions. We start with some thoughts about the very first session, which will provide the tutor with some idea of the background and aspirations of the students, as well as providing an opportunity to mould the working of the group for future sessions. It hardly needs to be stated that the first meeting of the group is of key importance - first impressions can be difficult to change!

Getting to Know the Group

This should be the first activity. Mastering names and faces as quickly as possible is both efficient and effective. The course team may help, for example by asking students to produce passport-sized photographs. One possibility is to ask each student to announce their name, their reasons for taking the course and any other background information that might be helpful. This has the bonus of ensuring that every member of the group contributes at the outset, and also ensures that the students in the group get to know each other. Once introductions have been made, it is good policy to address students by their first, or given, names; after all, that is how they will address each other. How students should address the tutor can be agreed between those involved.

It is important to remember, too, that some individuals will find problem-solving difficult and may require to work slowly and steadily, with only rare flashes of inspiration. Such students may well be taking the course by compulsion rather than choice, and some may initially feel resentment. The tutor will often start out with an advantage in not being one of those who selects and delivers the material in the first place, and so is not associated with their discomfort. It is hence particularly

important not to undermine that advantage by careless comments, forceful behaviour or demonstrations of easy brilliance, most especially at the outset.

Ground Rules

These are rules which all members of the group should accept in order to facilitate smooth running of the sessions. If the tutor's first session is at the start of the course, it is worth discussing with the group such rules and also their preferences for how the session should be run; it is good policy to review this at a later session. If the tutor is taking over mid-course, however, there is much to be gained by consulting the previous tutor (if possible) about the working of the group. Tutors should show from the outset that they are willing to be guided by reasonable requests.

In the case of first-year courses, students will usually be encountering tutorial sessions for the first time since leaving school, and the tutor may need to take a lead by suggesting enough basic rules and procedures to get the sessions off the ground. These can then be adapted and added to as the students gain more confidence. How to establish and maintain observation of ground rules, in general, is well-covered in chapter 3.

Can you think of additional ways in which you could help students get to know each other better?

Looking ahead to your tutorial classes, which 'ground rules' would you like the groups to consider adopting?

REGULAR SESSIONS

It is usual for the tutor's contract to specify that proper preparation must be carried out prior to the session. Otherwise it would be impossible to make good use of a period of, say, 50 minutes. We thus start with consideration of this vital matter, before discussing how its results are put into use during the class.

Tutor Preparation

The first step is to consult the course team's instructions and then study the material for the session, ensuring a reasonable degree of comfort with it. There may well be copies of guidance notes (as in *figure 2*) or model solutions to help in this. If there remains a problem, consultation of one of the set texts (mentioned in the course information sheets) may clear it up.

It is particularly important to pay attention to the context within which the material is presented. There are several factors to be considered:

- the depth of student knowledge;
- the sequencing of concepts - in pure mathematics, for example, it is possible either to define concept A and then prove result B, or to define concept B and then prove result A;
- any specialisation or simplification assumed by the lecturer concerned;
- which particular methods are advocated for solving a problem;
- notational or layout conventions;
- any possible relationship with material in students' other courses - this is particularly useful in 'service' courses, where it may help motivate the students and clarify the problem through discussing it in a familiar context.

Once the material and its context have been assimilated, any coursework should be 'marked' and commented on, as discussed later.

It is now time to prepare a programme for the session. There may be matters left over from the previous session to supply an opening line; alternatively, marking coursework may have suggested matters to be covered.

Flexibility is a key feature of any plan. Students may ask for a different topic to be covered; some new issue might arise in the process of dealing with the planned one; some material thought to be difficult may turn out to have been more easily grasped by the group. A careful consideration of time management is necessary, especially if faced with different demands from various members; 50 minutes for, say, ten individuals suggests a delicate balance must be struck between individual and group activity.

Finally, there are various potentially tricky situations which can arise in individual groups. Since these usually require careful handling they are explored in some detail in chapter 3.

It is difficult to estimate what can be achieved in the time allotted for a group whose characteristics you do not yet know. Try drawing up an outline plan for what you think may be a representative session. It may also be helpful to discuss this with a colleague who has previous tutoring experience.

Activities

Figure 3 set out some activities which could form part of a typical session: these are now expanded in the subsections below. Note that these are only indicative of what is possible; the list is neither complete nor compulsory. Further suggestions are offered by Hubbard[6].

Taking Attendance. If this is a requirement for the course, it can be done in conjunction with returning the coursework. In early sessions it can be made to appear part of the name-learning exercise. A middle course is to be steered; students should be aware that it is being done, but equally it should not be seen to be a big issue. If it is queried, it can be pointed out that attendance is a course requirement (probably set out in the course information sheet). It could even be added that it is a deliberate attempt to put a little pressure on students to attend, and so maintain an interest in the course, as experience normally shows a close correlation between those who fail to attend and those who fail the course!

Outlining Plan for Session. A brief outline should be given leaving, indeed encouraging, members of the group to comment. The plan should be updated in the light of replies.

Distributing Marked Work. Tutors should be careful not to use up time unduly in this way by labouring points already marked on the work; it is better to allow students to query written explanations they cannot follow. For some courses, the timetable can dictate a considerable time lag between the delivery of the material and the return of the coursework associated with it. In that case the course will have moved on to new material, and students should be coming to terms with the next assignment. Problems experienced by individuals should have been flagged on the scripts themselves. If remedial action is needed for most of the group, that should be included in the plan for the remainder of the session. The distribution of work is a useful activity for the very start of the session, since it does not compromise those who insist on flouting the 'ground rules' by late arrival.

Working on the Board. Tutor work on the board should normally be minimal, otherwise the tutorial becomes a lecture and the potential advantages of group work sessions are lost. The most useful role for the board is when there is a need to explain a point to the majority of students in the tutorial. Typical circumstances are:

- nearly everyone is baffled by some point in the set work (if this is a major problem, it probably affects all groups, and requires remedial action by the course team)
- in the tutorial someone raises a point which, when put to the group, is of interest to nearly everyone
- there is an opportunity to improvise something for the whole class (see below).

When a tutor does work on the board, it is good policy to ask the group questions, to see if they can fill in some of the points. This should avoid the students switching off on the grounds that the session has become another lecture. Sufficient time must be left for 'thinking', to avoid undue dependence on the tutor. It is important, however, to avoid embarrassing silences; if no-one replies, the tutor may have to provide the answer or perhaps start again with some further hints and suggestions. Tutors should not feel unduly apprehensive about this, since ability to judge when to intervene should develop with acquaintance with the students.

Grouping Students. This may be an effective way of getting students to help each other while the tutor is occupied elsewhere. One possibility is to arrange three students round a table or set of desks, with the tutor dropping in from time to time to fill the vacant slot. Some tutors like to carry a supply of triplicate paper, so that a copy of any written illustrations can be left with each student. Hubbard[6] discusses in detail various ways of grouping students.

In mathematics courses, some students may wish to work singly. It should be possible to allow both styles in a single tutorial, at least at the start of the course. Arranging small groups helps students get better acquainted. The use of such groups, however, must be carefully monitored. It is important to avoid the development of 'freeloaders' who take advantage of others' preparation. A group will work more effectively if all students have undertaken similar preparation; that can be used as a basis for allocation.

Students sitting side-by-side or in groups should be positively encouraged to discuss relevant points with one another. Sometimes their discussions may stray rather a long way from the tutorial topic, in which case a discreet reminder of the ground rules is called for. However, from the point of view of good atmosphere, it is probably sensible to allow a little 'social' conversation, but this needs to be kept within fairly tight limits.

Spreading Attention. If the group is working in subgroups, the tutor's attention is likely to be focussed on just part of the group or possibly one individual. At such times the rest of the tutorial group should be busy. To ensure this, early in the tutorial, perhaps after any overall business is dealt

with, it is necessary to make sure that everyone has something to work on. The lecturer may have suggested a particular problem, but as different students are likely to have reached different places on the sheets, it may not be advisable to be overly prescriptive about the work set for the tutorial hour. Any tutees currently unattended should be encouraged to call the tutor over if they encounter difficulties. If a point raised by a tutee looks interesting, he or she could be asked if it could be put to the whole group. If so, and it appears to be of wide-scale interest, do it on the board or, better still, get one of the group to try it.

Student Board Work. With some groups it may be possible to encourage a member to work on the board, which is an excellent exercise for all concerned. If a student is willing to respond to this, but then does something incorrectly, he or she should be let down gently. The tutor should try to find something positive in what has been done, but suggest that there was a point which had been overlooked; thanks should be expressed for the effort. If the work done by the student is correct, then praise is in order!

An alternative strategy is to ask the students at the board to act as a 'scribe' only. This takes much of the pressure off that student while forcing the others to communicate their contributions with clarity and precision.

It is, of course, necessary to take a great deal of care in choosing suitable topics for such work. Students are likely to resent it unless they have been given time for preparation. It would also be a mistake to involve only one or two students from the group. Whether or not to make regular use of this activity will depend on the students' backgrounds and abilities. It is likely to be inappropriate in a 'service' course or in a 'weak' group if students have been allocated by means of previous record. Suitable topics include:

- an exercise from the problem sheets or textbook
- an interesting point which has arisen in the group and could be 'researched' before the next meeting
- an investigation related to the course material
- a summary of the week's work from a student viewpoint.

Improvisation and Participation. Much of the best teaching that is done is improvised; it has a spontaneity that may be missing from formal lectures. If, before or during the tutorial, an idea appears which is relevant to the course (not necessarily all that directly: a different viewpoint, an overview, a neat method, a biographical detail, an interesting application, etc.) which can be put across without excessively labouring the point either on the board or verbally, it is worth seizing a suitable opportunity to air it. These excursions from the more routine work add precisely those bits of colour that will communicate enthusiasm for the subject to students. If some audience participation can be included, perhaps through selective questions, so much the better.

Tutors should not worry unduly if some technical device doesn't work when improvising. It is perfectly reasonable for tutors to be honest – admitting that the step eludes them, and promising to sort it out in the quiet of their own room; they must, however, make sure they do, in case the same question comes up the following week! Occasionally, students ask really searching questions, and if the answer can't be found immediately, this should be admitted with a promise to find out. It does students no harm to see that tutors are fallible, in fact it may well encourage them.

Winding up. It is good policy to summarise the session, feeding back any useful points which have arisen. If possible, pointers should be given for next week's preparation, this being a good way to encourage students to prepare for the sessions, in order to get the most out of them. This does depend on the course team briefing tutors on what is likely to be covered in the intervening period.

This is also a good time for the tutor to have a quiet word with any students who appear to need further help, directing them to course leader, lecturer or any drop-in consultancy service. Chapter 7, *Supporting and Advising Students*, gives further advice on this.

After the session has ended, tutors should complete any feedback sheet for the course team. They should report on any illnesses, either a retrospective report by the student, or a rumour from others in the group, as well as recording any student whose progress causes concern. Indeed, the type of information sheet exemplified in *figure 2*, if used, requires this.

Though few new tutors can be expected initially to have a wide range of finely tuned skills, all tutors will be able to handle some aspects of this complex task well.

Think through the various activities reviewed above. Are there any which you feel you will need to investigate in more depth? If so, you could consult an academic colleague or some of the books on tutoring[7].

EXAMPLE OF MARKED COURSEWORK

1. Express $\cos 3x$ in terms of $\cos x$.

$\cos 3x = \cos(2x + x)$

$= \cos 2x \cos x - \sin 2x \sin x$

$= (2\cos^2 x - 1)\cos x - \sin x \cos x \sin x$ X Oh dear! You made an excellent start but then forgot the 2 in $\sin 2x = 2 \sin x \cos x$.

$= (2\cos^2 x - 1)\cos x - \cos x(1 - \cos^2 x)$

$= 2\cos^3 x - \cos x - \cos x + \cos^3 x$

$= 3\cos^3 x - 2\cos x$ This is consistent with the slip above. Correct is $4\cos^3 x - 3\cos x$.

2. Find the derivative of $f(x) = \sin x^2$.

$f'(x) = \cos 2x$ X This is wrong, and it is difficult for me to say how you went wrong, since you give no explanation and show no working. This function requires use of the 'chain rule'. If that was indeed your approach, I'm afraid you used an incorrect version. Please ask me about this – the correct answer is $2x \cos x^2$.

3. Evaluate $I = \int x \ln x \, dx$.

Use integration by parts: Right idea, but unfortunately no progress is possible in this direction.

Let $u = x$, $\frac{dv}{dx} = \ln x$,

then $\frac{du}{dx} = 1$, $v = \frac{1}{2}(\ln x)^2$. X This is very wrong, but I suspect you were getting desperate!

$I = \left[\frac{1}{2}(\ln x)^2\right] - \int \frac{1}{2}(\ln x)^2 \, dx$

$= \frac{1}{2}(\ln x)^2 - \frac{1}{6}(\ln x)^3 + C$ X Correct structure, but a similar error in integration here.

Try using $u = \ln x$, $\frac{dv}{dx} = x$.

(In x cases have to be tackled in opposite way to other such examples.)

Try reworking your chosen method. You should now get $I = \frac{1}{2}x^2 \ln x - \frac{1}{4}x^2 + C$.

Figure 4

MARKING AND FEEDBACK

In what follows it is assumed that the main objectives in having work handed in for 'marking' are to provide diagnostic feedback and immediate help. Assessment towards the overall performance is assumed to be incidental to the tutor's role.

It has already been noted that problem-solving exercises tend to have an objective solution or a set of particularly creditable solutions. Students who have encountered difficulties with the tasks set may place 'blame' on the members of the course team who set the exercises and decided the mark scheme. This is an important observation, since it enables tutors to preserve their relationship with the students, even if assessment is involved. To maintain good relations, tutors should strive to put over the positive side of marking coursework,

backing this up with tactful and helpful comments on the script.

Comments such as 'rubbish' should be avoided; it is better to use phrases such as 'no real progress here'. Students should be encouraged by writing comments such as 'good work' or (if a serious error creeps in) 'right approach'. It is worth considering phrases such as 'Oh dear, you've gone off the rails here' or 'Pity, you had made a good start' to indicate the tutor is as disappointed as the student by the error made. Friendly phrases help encourage students to submit work and, even if it really is 'rubbish', it is better to receive something than nothing; otherwise tutorial help cannot be fine-tuned to the student's difficulties.

If a student shows a serious misunderstanding, even if the answer is nearly correct, adding 'Ask me about this' provides a starting point for tutorial help and discussion. References to course material such as printed notes or a set book, may be helpful to the student, but references to texts which are difficult to locate should be avoided, since that may waste students' time and result in lowered motivation. *Figure 4* provides an illustration of some of these points for mathematical coursework.

There are various common problems in dealing with coursework and these may be anticipated by the course team with specific advice. It is worthwhile thinking this matter through at an early stage, and tutors who are concerned by it should raise it at the briefing meeting with the course team. *Figure 5* draws together some potential tricky situations, with appropriate suggestions.

DEALING WITH TRICKY SITUATIONS

Poor presentation. The course team should provide advice about what is expected from the students, since their work ought to reflect the aims and objectives of the course. Matters such as the quality of layout and argument, will depend on the nature of the course - less emphasis on style and more on accuracy is common in 'service' courses. If such matters are important, tutors could perhaps comment separately and very briefly on coverage of technical points, presentation and logical argument. Again, a written comment such as 'Presentation - please ask' is liable to lead to a response in the tutorial where the tutor can elaborate.

Failure to complete parts of the assignment. Tutors should avoid simply spelling out a full solution, since that largely removes the point of the exercise. Instead they could try giving some broad hints, or even an outline of various key steps, followed by the final answer, if there is one.

'Cries for help'. Students may hand in blank or partially blank pages. In some cases this is a temporary problem caused by illness or finding that week's work particularly difficult. Help may be given as in the previous paragraph. In addition, it is important to enquire, tactfully, about the reason; if it was due to illness, the course leader may be able to excuse the student or to authorise late submission, if the grade is important. If, however, blank sheets or failure to hand in work is a regular problem, the student will need encouragement. Some suggestions for handling this are given in the next paragraph.

Late Work. Some students stop handing in work because they have dropped behind. Under these circumstances, tutors ought to be willing to accept from the student work covered earlier in the term, to look over it and credit it in much the same way as the set work. (It may be that the marks cannot count, if specimen solutions have been issued; on the whole it is better to accept it and sort out the grading issue later.) Tutors should stress to such students that when they stop handing in work, it becomes impossible to gauge progress or to give appropriate help. Slipping behind puts their studies into a risky situation, and they should take active steps to catch up as quickly as possible. Such students should be given constructive advice, even if it is only to seek help from their personal tutor or the course leader.

Cheating and Plagiarism. These are very delicate issues. There is often a fine line between collaborative work and copying. Students can often fail to attribute use of other peoples' work through carelessness rather than deception. Where to draw a line may depend on the course aims, and the only advice possible, from outside the context of the course, is to consult the course leader. It is quite likely that the course team will have anticipated the situation and issued guidelines as to what is acceptable and what action to take if there is prima facie evidence that they have been broken.

Figure 5

If marks are to be awarded, tutors must ensure they 'follow-through' from errors to see if subsequent work is accurate. They should not be petty – obvious slips of the pen should rarely be penalised. If an answer appears to be correct, but there is doubt about the method, tutors can add 'benefit of doubt' to ensure the student is aware of that and can seek advice if, indeed, he or she was uncertain about the method.

Even if the assessment counts for a great deal, this need not undermine teaching aspects. The above advice remains valid, but tutors may need to give more written explanation on how marks were awarded, to show fairness and retain the students' trust. Examples are:

- 'Two errors here', when an incorrect answer has incurred a double penalty
- 'This error has made the rest of the question trivial', when the tutor appears not to have given as many marks as 'following-through' would indicate
- 'I was prepared to ignore this first time, but I must penalise you now', when a student repeats an error which may have been a careless slip had it occurred once only.

Do you feel comfortable about the level of detail you should give in response to missing or wholly inaccurate coursework assignments? If not, you could consult a colleague with previous tutoring experience in your discipline.

CONCLUSION

The task of tutoring in a problem-solving environment is a significant and stimulating challenge. To diagnose students' difficulties, and to help overcome them, often requires the tutor to replicate their thought processes. Each student's mind is unique, and the successful tutor will develop a battery of techniques to use on the diversity of issues which arise. The material in this chapter undoubtedly covers only a proportion of successful tutoring strategies.

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